FREE SAMPLE

GRADE 12 MATHEMATICS · CAPS

See before you buy

A real page from every Top Grades resource

TOP GRADES
TUTORING

What is in this sample

1	**Summary Pack**	1 of 12 one-page chapter summaries
2	**Theory Pack**	the start of Chapter 3: Functions
3	**Practice Exams**	cover and Question 1 of Set 1, Paper 1
4	**Explained Memo**	Question 1 memo and the scorecard
5	**Mastery Worksheets**	the first two pages of Worksheet 1

TOP GRADES TUTORING

Get the full packs

10 original exam papers with explained memos, 12 theory chapters, 5 worksheets and 12 summary sheets, in English and Afrikaans. Available at topgrades-tutoring.com.

SUMMARY · CHAPTER 3 · PAPER 1 about 35 marks

Functions & inverses

Parabola, hyperbola, exponential: sketch, read and invert

TOP GRADES TUTORING

Parabola

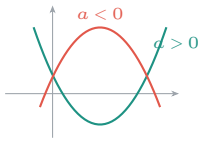
$y = a(x+p)^2 + q$

TP $(-p; q)$ · axis $x = -p$

$y = ax^2 + bx + c$: $x = -\dfrac{b}{2a}$

roots form:

$y = a(x - x_1)(x - x_2)$



Hyperbola

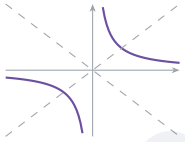
$y = \dfrac{a}{x+p} + q$

asymptotes $x = -p$, $y = q$

axes of symmetry:

$y = \pm(x+p) + q$

$a > 0$: quadrants I & III



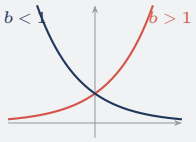
Exponential

$y = a \cdot b^{x+p} + q$

asymptote $y = q$

$b > 1$ increasing · $0 < b < 1$ decreasing

$y = b^x$ passes through $(0; 1)$



What a, p and q do

$x + p$	shift **left** by p (opposite sign!)
$+q$	shift **up** by q
$a < 0$	reflect in the x-axis
$\lvert a \rvert$ big	steeper / narrower

1 Sketch any graph in 5 steps

1. shape (a)
2. y-intercept ($x = 0$)
3. x-intercepts ($y = 0$)
4. TP or asymptotes
5. label every point and asymptote

Equation from a graph

TP given → $y = a(x+p)^2 + q$

roots given → $y = a(x - x_1)(x - x_2)$

asymptotes given → fill in p, q first

then substitute **one more point** to find a

Read the graph: translate

$f(x) > 0$	above the x-axis
$f(x) \geq g(x)$	f on or above g
$f(x) \cdot g(x) < 0$	opposite sides of x-axis
$f'(x) > 0$	f increasing
$f(x) = k$, 2 roots	line $y = k$ cuts twice

Answers are always **x-values**. Find the intersections first.

Vertical distance

length = **top graph** − **bottom graph**

$PQ = f(x) - g(x)$, then max at $x = -\frac{b}{2a}$ of the new parabola.

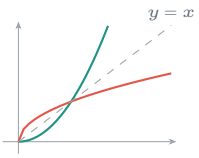
Inverse functions

1. swap x and y
2. make y the subject

reflection in $y = x$

$(a; b)$ on f → $(b; a)$ on f^{-1}

domain of f = range of f^{-1}



Is the inverse a function?

Only if f is **one-to-one**. A parabola is not: restrict it to $x \geq 0$ or $x \leq 0$ first.

Exam check: horizontal line test.

e.g. Exam-style example

$h(x) = \dfrac{2}{x-1} + 3$: asymptotes $x = 1$, $y = 3$

x-intercept: $\frac{2}{x-1} = -3 \Rightarrow x = \frac{1}{3}$ · axis: $y = -(x-1) + 3$

Exam ready? I can...

- ☐ sketch all three graphs
- ☐ find a, p, q from a sketch
- ☐ read inequalities off graphs
- ☐ find a vertical distance
- ☐ find and sketch f^{-1}
- ☐ restrict a parabola

Mark traps

- ✗ $(x+2)$ means LEFT 2, not right.
- ✗ A range is written with y, not x.
- ✗ Brackets: $f(x) - g(x) = \cdots - (x+2)$.
- ✗ Never include an asymptote in an answer.
- ✗ $f^{-1}(x) \neq \dfrac{1}{f(x)}$.
- ✗ Label intercepts and asymptotes on sketches.

CHAPTER 3 · PAPER 1

Functions & inverses

Graphs, their features and inverses

Grade 11 graphs plus Grade 12 inverses · about 35 marks in Paper 1

3.1 The three graphs | 3.2 Sketching | 3.3 Reading graphs | 3.4 Inverses

TOP GRADES TUTORING

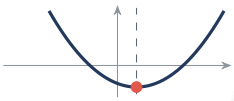
3.1 The three graphs

- A **function** gives every x-value **exactly one** y-value. Test: no vertical line may cut the graph twice.
- **Domain** = all the x-values. **Range** = all the y-values.

Know these three families

PARABOLA

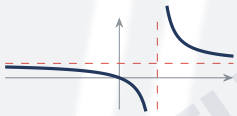
$y = a(x+p)^2 + q$



Turning point: $(-p\,;\,q)$
Axis: $x = -p$
$a > 0$: minimum · $a < 0$: maximum
For $ax^2 + bx + c$: $x = -\frac{b}{2a}$

HYPERBOLA

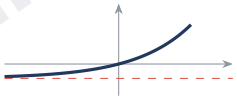
$y = \frac{a}{x+p} + q$



Asymptotes: $x = -p$, $y = q$
Axes of symmetry:
$y = (x+p) + q$
$y = -(x+p) + q$
$a > 0$: quadrants 1, 3 · $a < 0$: 2, 4

EXPONENTIAL

$y = a \cdot b^{x+p} + q$



Asymptote: $y = q$
$b > 1$: increasing
$0 < b < 1$: decreasing
$a < 0$: flipped over the asymptote
Range: $y > q$ (if $a > 0$)

WATCH OUT

- p works **opposite** to how it looks: $y = (x+2)^2$ moves **2 left**, not right. Turning point $(-2;0)$.
- Range is about y, domain is about x. Write them with the right letter: $y \geq -1$, not $x \geq -1$.

IN A NUTSHELL: I CAN…

- ☐ Use the vertical line test
- ☐ Name the asymptotes of a hyperbola
- ☐ Give the turning point and axis of a parabola
- ☐ Describe the effect of a, p and q

3.2 Sketching and finding equations

What do a, p and q do?

p · **left/right**	$(x+p)$ moves the graph **left** by p; $(x-p)$ moves it **right**.
q · **up/down**	$+q$ moves the graph **up**; $-q$ moves it **down**.
a · **shape**	Sign: which way it opens / which quadrants. Size: how steep or wide.

How to sketch any graph

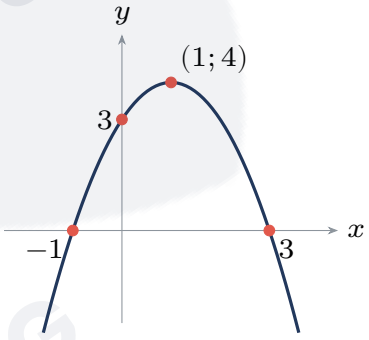
1 Shape (sign of a) → **2** y-intercept ($x=0$) → **3** x-intercepts ($y=0$) → **4** TP or asymptotes

label every point you worked out!

EXAMPLE 3.1 Sketch $f(x) = -(x-1)^2 + 4$

- $a = -1 < 0$ → frown, so a **maximum**.
- Turning point: $(1\,;\,4)$
- y-intercept: $f(0) = -(0-1)^2 + 4 = 3$
- x-intercepts: $-(x-1)^2 + 4 = 0$

$$(x-1)^2 = 4$$
$$x - 1 = \pm 2$$
$$x = 3 \quad \text{or} \quad x = -1$$



Finding the equation from a graph

PARABOLA

Given the **TP**?

$y = a(x+p)^2 + q$

Given **x-intercepts**?

$y = a(x-x_1)(x-x_2)$

Then substitute another point for a.

HYPERBOLA

Asymptotes give p and q.

Substitute a point to find a.

EXPONENTIAL

Asymptote gives q.

Substitute points to find a and b.

TOP GRADES TUTORING

TOP GRADES TUTORING

GRADE 12

PRACTICE EXAMINATION

MATHEMATICS P1

SET 1

QUESTION PAPER

MARKS: 150 **TIME: 3 hours**

This question paper consists of 8 pages, including the information sheet.

TOP GRADES TUTORING

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 2x - 7 = 0$ (correct to TWO decimal places) (4)

1.1.2 $x^2 + 3x \geq 10$ (4)

1.1.3 $\sqrt{2x + 7} = x + 2$ (4)

1.2 Solve for x and y simultaneously:

$$y - 2x = 1 \qquad \text{and} \qquad x^2 + y^2 - xy = 7$$

(6)

1.3 Determine the value of k for which $kx^2 - 6x + 3 = 0$ has two equal roots. (3)

[21]

QUESTION 2

2.1 Consider the quadratic number pattern: $5\ ;\ 8\ ;\ 15\ ;\ 26\ ;\ ...$

2.1.1 Write down the next term of the pattern. (1)

2.1.2 Determine an expression for the nth term of the pattern. (4)

2.1.3 Which term of the pattern is equal to 1 181? (4)

2.2 The sum of the first n terms of an arithmetic series is given by $S_n = 3n^2 + 2n$.

2.2.1 Write down the first term of the series. (1)

2.2.2 Calculate the 10th term of the series. (3)

2.3 Consider the geometric series: $48 + 24 + 12 + ...$

2.3.1 Calculate the sum of the first 8 terms of the series. (3)

2.3.2 Explain why this series converges, and calculate its sum to infinity. (3)

2.4 Evaluate: $\sum_{k=1}^{30}(4k - 7)$ (3)

[22]

TOP GRADES TUTORING

QUESTION 1

1.1.1

Level 2 · Routine procedure

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-7)}}{2(3)} \checkmark \qquad = \frac{-2 \pm \sqrt{88}}{6} \checkmark$$

$$x = 1{,}23 \checkmark \quad \text{or} \quad x = -1{,}90 \checkmark$$

Why: "Two decimal places" tells you it won't factorise, so use the formula. Writing $a = 3$, $b = 2$, $c = -7$ first avoids sign errors under the root.

Alternative method: Completing the square: $x^2 + \frac{2}{3}x = \frac{7}{3}$, so $\left(x + \frac{1}{3}\right)^2 = \frac{7}{3} + \frac{1}{9} = \frac{22}{9}$. Then $x = -\frac{1}{3} \pm \frac{\sqrt{22}}{3}$, giving $x = 1{,}23$ or $x = -1{,}90$.

(4)

1.1.2

Level 2 · Routine procedure

$$x^2 + 3x - 10 \geq 0 \checkmark \qquad (x + 5)(x - 2) \geq 0 \checkmark$$

$$x \leq -5 \checkmark \quad \text{or} \quad x \geq 2 \checkmark$$

Why: The parabola $y = (x + 5)(x - 2)$ smiles, so it is *above* the x-axis *outside* the critical values -5 and 2. "$\geq$" includes the critical values.

Alternative method: Sign table (number line): mark -5 and 2. Test $x = -6$: $(-)(-) = +$; $x = 0$: $(+)(-) = -$; $x = 3$: $(+)(+) = +$. Choose the regions where the product is ≥ 0.

(4)

1.1.3

Level 3 · Complex procedure

$$2x + 7 = x^2 + 4x + 4 \checkmark \qquad x^2 + 2x - 3 = 0 \qquad (x + 3)(x - 1) = 0 \checkmark$$

$$x = -3 \text{ or } x = 1 \qquad \text{Check } x = -3: \ \sqrt{1} = 1 \neq -1, \text{ reject} \checkmark \qquad x = 1 \checkmark$$

Why: Squaring both sides can create a fake answer. A square root is never negative, so the right-hand side $x + 2$ must be ≥ 0. That rules out $x = -3$.

Marks Leak: Learners often stop at "$x = -3$ or $x = 1$". Without checking, the final answer mark is lost. Always substitute back into the *original* equation.

(4)

MY SCORECARD: WHERE DID I LOSE MARKS?

Mark your paper with this memo, then fill in your marks for each topic. Any topic below 60 % is your priority: do its **next step** before you write the next paper.

Topic	Q	Out of	My mark	%	Next step if below 60 %
Algebra, equations & inequalities	1	21			Theory Pack Ch 1: quadratic equations, inequalities, simultaneous equations, nature of roots
Sequences & series	2	22			Theory Pack Ch 2 (all topics)
Parabola & straight line	3	18			Theory Pack Ch 3: the three graphs, reading graphs. Then Worksheet 1
Hyperbola, exponential & log graphs	4	15			Theory Pack Ch 3 (inverses) and Ch 4. Then Worksheet 1
Finance	5	17			Theory Pack Ch 5: interest rates, present value annuities. Then Worksheet 2
First principles, rules & tangents	6	16			Theory Pack Ch 8: first principles, rules, equation of a tangent
Cubic graphs	7	16			Theory Pack Ch 7 and Ch 8 (cubic graphs). Then Worksheet 3
Optimisation & rates of change	8	11			Theory Pack Ch 8: optimisation. Then Worksheet 3
Probability & counting	9	14			Theory Pack Ch 12: probability rules, counting principle
TOTAL		**150**			

MARKS BY LEVEL

Each question in this memo is labelled with a level. The level describes the *kind of thinking* a question needs. It says nothing about how clever you are: Level 3 and 4 skills grow with practice, just like the others.

Level	Out of	My mark
Level 1 · Knowledge (formulas, facts, reading off)	22	
Level 2 · Routine procedure (standard methods)	69	
Level 3 · Complex procedure (several steps or linking ideas)	41	
Level 4 · Problem solving (unfamiliar or non-routine)	18	

Top Grades Tutoring learners: bring this page to your next session, so we can start with the topic that will give you the most marks back.

TOP GRADES TUTORING

MASTERY WORKSHEET 1 · PAPER 1

Functions, inverses & logs

Reading graphs, finding equations and inverses

The graph skills that come up in almost every Paper 1

TOP GRADES TUTORING

1 Toolkit | 2 Worked examples | 3 Your turn | 4 Mistakes | 5 Answers

HOW TO USE THIS WORKSHEET

1 Read the toolkit and the worked examples. 2 Try every question on your own, with a timer. 3 Mark with the answers at the back: they explain *why*, not just *what*. 4 Redo every question you got wrong a week later.

A The toolkit

PARABOLA

$y = a(x-p)^2 + q$ (turning point $(p;q)$)

$y = a(x-x_1)(x-x_2)$ (roots x_1, x_2)

axis of symmetry halfway between the roots

HYPERBOLA

$$y = \frac{a}{x-p} + q$$

asymptotes $x = p$ and $y = q$; axes of symmetry $y = \pm(x-p) + q$

EXPONENTIAL

$y = a \cdot b^{x-p} + q$

asymptote $y = q$; $b > 1$ increases, $0 < b < 1$ decreases

LOGARITHM

$y = \log_b x \iff x = b^y$

inverse of $y = b^x$; domain $x > 0$; x-intercept $(1;0)$

FINDING AN INVERSE IN 3 STEPS

1 Swap x and y. 2 Make y the subject. 3 Restrict the domain or choose the sign if the original was not one-to-one.

The graph of f^{-1} is the reflection of f in the line $y = x$, so every point $(a;b)$ on f becomes $(b;a)$ on f^{-1}.

READING INEQUALITIES OFF A SKETCH

- $f(x) > g(x)$ means: where is the graph of f **above** the graph of g? The answer is always a set of x**-values**.
- $f(x) > 0$ means above the x-axis; $f(x) \cdot g(x) < 0$ means the two graphs are on **opposite sides** of the x-axis.
- Find the intersection points first. They are the "borders" of your answer.

B Worked examples

EXAMPLE W1.1 Given $h(x) = \dfrac{4}{x-2} + 1$ and the line $y = x - 1$.

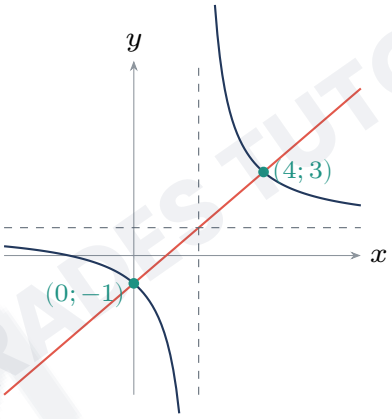
(a) Asymptotes: $x = 2$ and $y = 1$

read p and q straight off

(b) Intercepts:

y-int: $h(0) = \frac{4}{-2} + 1 = -1$

x-int: $\frac{4}{x-2} = -1 \Rightarrow x = -2$



(c) Is $y = x - 1$ an axis of symmetry?

It passes through $(2; 1)$, where the asymptotes cross, and has gradient 1. Yes.

(d) For which x is $h(x) \geq x - 1$?

$$\frac{4}{x-2} + 1 = x - 1 \Rightarrow 4 = (x-2)^2 \Rightarrow x = 0 \text{ or } x = 4$$

intersections first

$h(x) \geq x - 1$: $x \leq 0$ or $2 < x \leq 4$

$x = 2$ excluded: asymptote

Check with a test value: at $x = -2$, $h = 0$ and the line is -3, so h is above. At $x = 5$, $h \approx 2{,}3$ and the line is 4, so h is below.

EXAMPLE W1.2 Given $f(x) = \left(\frac{1}{3}\right)^x$. Find f^{-1} and solve $f^{-1}(x) > -1$.

$x = \left(\frac{1}{3}\right)^y$ *step 1: swap*

$y = \log_{\frac{1}{3}} x$ *step 2: definition of a log*

$f^{-1}(x) = -1$ when $x = \left(\frac{1}{3}\right)^{-1} = 3$. The base is between 0 and 1, so f^{-1} is a **decreasing** graph: it is above -1 to the *left* of $x = 3$, but only where it exists ($x > 0$).

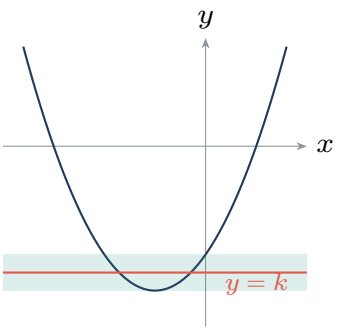
$$0 < x < 3$$

EXAMPLE W1.3 Given $f(x) = x^2 + 2x - 3$. For which values of k will $f(x) = k$ have two unequal *negative* roots?

- Turning point: $x = -1$, $f(-1) = -4$.

  the lowest value
- y-intercept: $f(0) = -3$.
- Draw horizontal lines $y = k$: they cut both arms to the **left** of the y-axis only between -4 and -3.

$$-4 < k < -3$$



TOP GRADES TUTORING